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A UNITED DESCRIPTION FOR DARK MATTER AND DARK ENERGY

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Abstract. *In this paper, we show a unifying description to the dark matter and dark energy. This description does not demand dark energy with the anti-gravitational property. It also points out a lower limit of the average mass of the particles of cosmological energy (ordinary matter, dark matter and dark energy particles)* $\overline{m} \gg 54\text{eV}$*. The coincident problem between the density of dark energy and one of matter is a clear fact.*

I. INTRODUCTION

Recent astro-physical observations show that the Universe consists of 4 % ordinary matter, 23 % dark matter, 73 % dark energy [1].

The existence of dark matter has pointed out firstly by Jan Oort (1930) and Frizt Zwicky (1933) [2] based on the studies of the rotation curves of galaxies and galactic clusters. The main candidates for dark matter are MACHOs (Massive Astrophysical Compact Halo Objects) and WIMPs (Weakly Interacting Massive Particles).

Dark energy is an unknown form of energy with negative pressure. Nowadays it causes the accelerating expansion of the Universe.

The existence of dark energy has been pointed out directly by two independent groups based on Supernovae (SNe) type Ia observations [3, 4] and also indirectly been suggested by independent studies based on fluctuations of the 3K relic radiation [5], large scale structure [6], age estimates of globular clusters, old high red-shift objects [7], as well as by the X-ray data from galaxy clusters [8].

Nowadays, there are many other candidates for the dark energy [9]:

1. A cosmological constant Λ;
2. A $\Lambda(t)$ - term, or a decaying vacuum energy density;
3. A relic scalar field (SF) slowly rolling down its potential;
4. X-matter, an extra component characterized by an equation of state $p_X = \omega\rho_X,\ -1 \leq \omega < 0$;
5. A Chaplygin - type gas whose equation of state is given by $p = A/\rho^{\alpha},\ 0 \leq \alpha \leq 1$ where A is a positive constant.

It is widely known that the main distinction between the pressure-less CDM and dark energy is that the former agglomerates at small scales whereas the dark energy is a smooth component in the Universe. Such properties seems to be directly linked to the equation of state of both components. Recently, the idea of a unified description for

CDM and dark energy scenarios has received much attention. For example, Wetterich [10] suggested that dark matter might consist of quintessence lumps, Kasuya [11] showed that spintessence type scenarios are generally unstable to formation of Q balls which behave as pressure-less matter. More recently, Padmanabhan and Choudhury [12] investigated such a possibility trough a string theory motivated tachyonic field. Kamenshchik et al [13], Billic et al [14], Beto et al also suggested the unification which refers to an exotic fluid, the Chaplygin type gas, whose equation of state is: $p = A/\rho^{\alpha}$. For $\alpha < 1$, this equation constitutes a generalization of the original Chaplygin gas equation of state [15], for $\alpha = 0$, model behaves as ΛCDM.

An another way of a unified description of dark matter and dark energy is K-essence models. The idea of K- essence was first introduced as a possible model for inflation [16, 17]. Later it was noted that K-essence can also yield interesting models for the dark energy [18, 19, 20, 21]. It is possible to construct a particular interesting class of such models in which the K-essence energy density tracts the radiation energy density during the radiation-dominated era, but then evolves toward a constant density dark energy component during the matter dominated era [5, 6]. In this class of models, the coincidence problem is resolved by linking the onset of dark energy domination to the epoch of equal matter and radiation.

R. J. Scherrer [22] reexamined a particularly simple class of K-essence, in which the lagrangian contains only a kinetic factor, i.e. a function of the derivatives of the scalar field, and does not depend explicitly on the field. He also examines such models in the generic case. These models naturally produce a density which scales like the sum of a non-relativistic dust component with the equation of state $\omega = 0$ and a cosmological - constant -like component $\omega = -1$. The other distinguishing characteristic of these models is that they generically produce a low sound speed, allowing the "dust" component to cluster as dark matter.

In this paper, based on the vector model of gravitational field we also introduce a united description for dark matter and dark energy. This description does not require the anti- gravitation property for dark energy. It also point out the truncation of dark matter halos and a lower limit for the average mass of dark matter and dark energy particles. The coincident problem between the density of dark energy and one of matter is a clear fact.

II. AN APPROACH TO THE DARK MATTER AND DARK ENERGY PROBLEMS

From result in the paper [23], we have known that the density of the cosmological energy (ordinary matter, dark matter and dark energy) dilutes in the form $\rho \propto R^{-2}$ in the vector model of gravitational field. Thus, basing on the dilution of the density, the cosmological energy is like the dust matter ($\rho_m \propto R^{-3}$) than the radiation energy ($\rho_R \propto R^{-4}$). Because of this fact, we assume that the classical Bolzmann distribution can be used to describe the distribution of the cosmological energy around galaxies and galactic clusters.

We consider a galaxy with the gravitational mass M_g, it is in a sea of the cosmological energy. We investigate the gravitational field at a point A in this sea.

Call N_0 is the density of cosmological energy particles at a very distance point from the galaxy when $\varphi_g = 0$.

Call φ_g is the gravitational potential at A. When we assume that the classical Boltzmann distribution can be applied to the cosmological energy, we have the density of particles at A

$$N = N_0 \exp\left(-\frac{m_g \varphi_g}{kT}\right) \tag{1}$$

Here m_g is the gravitational mass of a particle. Thus density of gravitational mass at A is

$$\rho_g = N m_g = m_g N_0 \exp\left(-\frac{m_g \varphi_g}{kT}\right) \tag{2}$$

Here m_g is the gravitational mass of particle, T is the absolute temperature of the particle gas. At a remote distance from the galaxy, the kinetic energy of particle is very larger than its potential energy. We suppose that

$$m_g \varphi_g \ll kT \tag{3}$$

We have

$$\exp\left(-\frac{m_g \varphi_g}{kT}\right) \approx 1 - \frac{m_g \varphi_g}{kT} \tag{4}$$

From (2)

$$\rho_g = m_g N_0 \left(1 - \frac{m_g \varphi_g}{kT}\right) \tag{5}$$

$$= m_g N_0 - \frac{m_g^2 N_0 \varphi_g}{kT} \tag{6}$$

We recall the 3th equation of the system of non-relativistic equations [24]

$$\nabla \overrightarrow{E}_g = -\frac{\rho_g}{\varepsilon_g} \tag{7}$$

Notice that $\overrightarrow{D}_g = \varepsilon_g \overrightarrow{E}_g$ [24], $G = 1/4\pi\varepsilon_g$ and $\overrightarrow{E}_g = -\nabla\varphi_g$ due to $\overrightarrow{A}_g = 0$, thus

$$\nabla^2 \varphi_g = \frac{\rho_g}{\varepsilon_g} \tag{8}$$

Substituting (6) into (8), we have

$$\nabla^2 \varphi_g = \frac{m_g N_0}{\varepsilon_g} - \frac{m_g^2 N_0}{\varepsilon_g kT} \varphi_g \tag{9}$$

We rewrite (9) in the following form

$$\nabla^2 \varphi_g = a - b^2 \varphi_g \tag{10}$$

Here

$$a \equiv \frac{m_g N_0}{\varepsilon_g} \tag{11}$$

$$b^2 \equiv \frac{m_g^2 N_0}{\varepsilon_g kT} \tag{12}$$

Assuming that the cosmological energy particles distribute symmetrically around galaxy M_g, we can rewrite (10) in the following form:

$$\frac{1}{r}\frac{d^2}{dr^2}(r\varphi_g) = -b^2\varphi_g + a \tag{13}$$

We seek the general root of equation (13). The general root of homogeneous equation (13)

$$\frac{1}{r}\frac{d^2}{dr^2}(r\varphi_g) = -b^2\varphi_g \tag{14}$$

is

$$\varphi_{g0} = \frac{1}{r}\left(C_1 e^{-ibr} + C_2 e^{+ibr}\right) \tag{15}$$

A special root of inhomogeneous equation (13) is

$$\varphi_{g1} = \frac{1}{r}\left(e^{-ibr} + e^{+ibr}\right) + \frac{a}{b^2} \tag{16}$$

Thus, the general root of inhomogeneous equation (13) is

$$\varphi_g = \varphi_{g0} + \varphi_{g1} = \frac{1}{r}\left(C_1 e^{-ibr} + C_2 e^{+ibr}\right) + \frac{1}{r}\left(e^{-ibr} + e^{+ibr}\right) + \frac{a}{b^2} \tag{17}$$

Due to φ_g and a/b^2 are real, so $C_1 = C_2$ and they are real or $C_1 = -C_2$ and they are purely imaginary. We require that when $r \to 0$, we obtain Newtonian limit for the gravitational potential, i.e.

$$\varphi_g \to \varphi_g = -\frac{GM_g}{r} + \text{constant} \tag{18}$$

When $r \to 0$, φ_g in (17) becomes

$$\varphi_g = \frac{1}{r}(C_1 + C_2) + \frac{2}{r} + \frac{a}{b^2} \tag{19}$$

In the case when $C_1 = C_2$, from (18) and (19), we obtain

$$C_1 + 1 = C_2 + 1 = -\frac{GM_g}{2} \tag{20}$$

and $a/b^2 = \text{constant}$. In the case when $C_1 = -C_2$, we don't obtain the classical Newtonian limit. We shall discuss this case in a different paper. Thus the general root of (13) is

$$\varphi_g = -\frac{GM_g}{r}\left(e^{-ibr} + e^{ibr}\right) + \frac{a}{b^2} = -\frac{GM_g}{r}\cos br + \frac{a}{b^2} \tag{21}$$

We also obtain the gravitational field around galaxy M_g when the cosmological energy presents as follows

$$E_g = -\text{grad}\varphi_g = -\frac{GM_g b}{r}\sin br - \frac{GM_g}{r^2}\cos br \tag{22}$$

Finally, the gravitational force acts on a star m_{g1} which moves in this gravitational field as follows

$$F_g = m_{g1}E_g = -\frac{GM_g b m_{g1}}{r}\sin br - \frac{GM_g m_{g1}}{r^2}\cos br \tag{23}$$

We rewrite (23) in the following form

$$F_g = F_V + F_N \tag{24}$$

with

$$F_V \equiv -\frac{GM_g b m_{g1}}{r}\sin br \quad \text{(the vacuum force)} \tag{25}$$

$$F_N \equiv -\frac{GM_g m_{g1}}{r^2}\cos br \quad \text{(the Newtonian force)} \tag{26}$$

We consider now the correlation between F_V and F_N when r varies from 0 to ∞.

1. Region 1 (Newtonian region): when $br \ll 1$ we have $\sin br \approx 0,\ \cos br \approx 1$ so $F_V \ll F_N$ we see that

$$F_g = F_N = -\frac{GM_g m_{g1}}{r^2} \tag{27}$$

We return the Newtonian limit.

2. Region 2 (Region of dark matter): when $br = \frac{\pi}{2} \pm \varepsilon$ we have $\sin br \approx 1,\ \cos br \approx 0$ so $F_V \gg F_N$ Therefore

$$F_g = F_V = -\frac{GM_g m_{g1} b}{r} \tag{28}$$

If we investigate now the motion of a star in this region, we have

$$m_{i1}\frac{v^2}{r} = \frac{GM_g m_{g1} b}{r} \tag{29}$$

Because $m_{i1} \cong m_{g1}$ therefore

$$v^2 = GM_g b \tag{30}$$

i.e. v is independent of r. When taking into account of the term $sinbr$, we have $v^2 = GM_g b \sin br$ or

$$v = (GM_g b \sin br)^{1/2} \text{ with } \frac{\pi}{2} - \varepsilon < br < \frac{\pi}{2} + \varepsilon \tag{31}$$

We express the results on the figure (1).

3. Region 3 (Region of dark energy): when $|\cos br| > \sin br$ or $br > \pi$. F_N changes sign and becomes repulsive force and $|F_N| > F_V$ or both F_N and F_V become repulsive forces. A star or other galaxy into this region would be repulsed away and accelerated. Perhaps acceleration of the Universe on large distances occurs when galaxies are in this region.
4. Region 4 (large attractive region): when br is very large. F_V and F_N change signs again and become attractive forces. We show these regions on the figure (2).

III. THE EVALUATION OF b AND THE AVERAGE MASS OF THE PARTICLES OF COSMOLOGICAL ENERGY

Now we evaluate the value of b. We examine the rotation curve of Milky Way . The Sun which is in the region of dark matter has the rotation velocity around the Milky Way about 200 km/s, the Milky Way's mass is about $10^{11}M_{\text{sun}}$. From (30), we have

$$b = \frac{v^2}{GM_g} \tag{32}$$

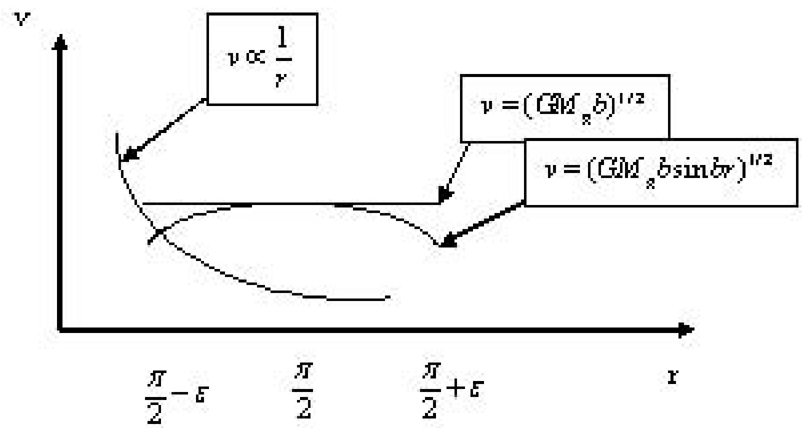

Fig. 1. The dependence of star velocity on the distance r from the center of galaxies.

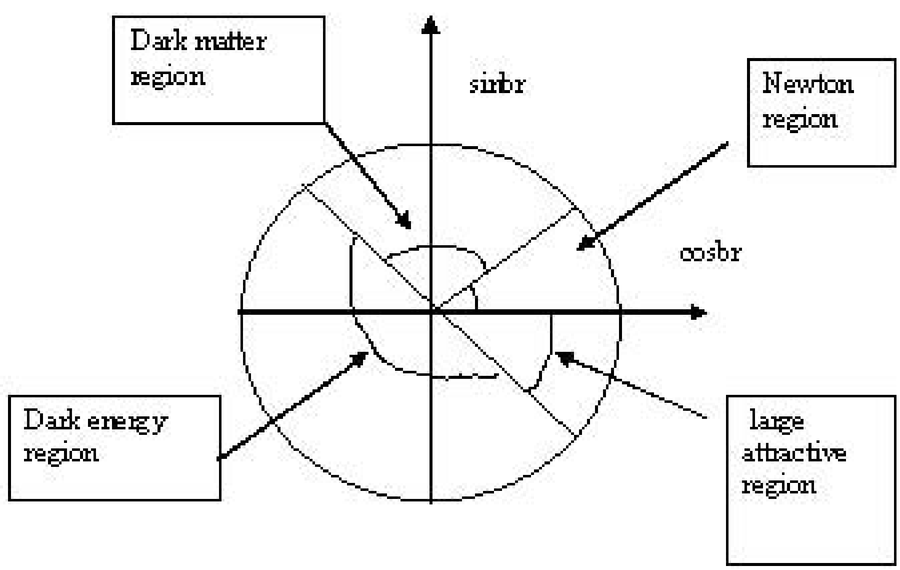

Fig. 2. Space regions around galaxy when r increases.

where $v = 2 \times 10^5$ m/s and $M_g \sim 10^{11} \times 2 \times 10^{30}$ kg. We get

$$b \sim 3 \times 10^{-21} \text{ m}^{-1} \tag{33}$$

We also find that radii of planets in the solar system are in the Newtonian region. Indeed, when we choose $r_{\max} = r_{\text{Pluto}} = 5500.10^9$ m we obtain

$$br_{\text{Pluto}} = 3 \times 10^{-21} \times 5500 \times 10^9 = 165 \times 10^{-10} << 1 \tag{34}$$

We have also known that almost masses of galaxies are about 10^{11} M_{sun}, their velocities in rotation curves are about 150 km/s $\rightarrow$ 300 km/s [2, 25, 26, 27], therefore value of b in (33) is true.

We evaluate the first dark matter region of galaxies. From Fig.(2) and formulas (31) we find that the region of dark matter starts at the distance so that $F_V \gg F_N$ and ends as $br_{\max} = \pi$ or $r_{\max} \sim 1.04 \times 10^{21}$ m ~ 33.8 kpc.

We evaluate the mean mass of the cosmological energy particles in this approach. It is known that particles with integral spins obey the Bose - Einstein statistics , particles with half- integral spins obey the Fermi-Dirac statistics. However, when the particle gas satisfies the non- degeneracy condition [28, 29]

$$\frac{n_0 h^3}{(2\pi mkT)^{3/2}} \ll 1 \tag{35}$$

These two statistics lead to the classical Bolzmann statistics. Here n_0 is the particle density, $h = 6.63 \times 10^{-34}$ J.s is the Planck constant, $k = 1.38 \times 10^{-23}$ J.K^{-1} is the Bolzmann constant, m is the particle mass, T is the absolute temperature of the particle gas. We can substitute $n_0 = \rho/m$ with ρ is the mass density of particles, (35) becomes

$$\frac{\rho h^3}{(2\pi kT)^{3/2} m^{5/2}} \ll 1 \tag{36}$$

But we have $\rho_0 = m_g N_0 \leq \rho$ (because ρ_0 is the density at the points which are very distance from the galaxy). Therefore

$$\frac{\rho_0 h^3}{(2\pi kT)^{3/2} m^{5/2}} \ll 1 \tag{37}$$

We also recall (12)

$$b^2 = \frac{m_g^2 N_0}{\varepsilon_g kT} \tag{38}$$

$$= \frac{m_g \rho_0}{\varepsilon_g kT} \tag{39}$$

Therefore

$$kT = \frac{m_g \rho_0}{b^2 \varepsilon_g} = \frac{m_g \rho_0 4\pi G}{b^2} \tag{40}$$

Substitute (40) into (37), we have:

$$\frac{h^3 b^3}{(8\pi^2 G)^{3/2} \rho_0^{1/2} m_g^4} \ll 1 \tag{41}$$

or

$$\frac{(hb)^{3/4}}{(8\pi^2 G)^{3/8}\rho_0^{1/8}} \ll m_g \tag{42}$$

If we choose $\rho_0 \sim 10^{-29}$ g/cm$^3 \sim 10^{-26}$kg/m^3, we have:

$$m_g \gg 1.3 \times 10^{-34} \text{ kg} \sim 54 \text{ eV} \tag{43}$$

We also remark that b can has different values in clusters of galaxies and superclusters of galaxies by (12) and (30).

A remarkable point of this approach is that it do not demand dark energy with anti-gravitational property.

The coincidence problem between the density of dark energy and the density of matter is a clear fact because there is no the distinction between ordinary matter , dark matter and dark energy in this approach.

IV. CONCLUSION

In this paper, we have introduced a united description for dark matter and dark energy. we have obtained a modified expression for the gravitational force and have found a lower limit of the average mass of the particles of cosmological energy.

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